PRE-BOARD EXAMINATION – 2023

Std. 12 **MATHEMATICS** **Time : 3 hrs.**

20-12-2023 **Max. Marks : 80**

GENERAL INSTRUCTIONS:

i) All questions are compulsory.
ii) This Question Paper is divided into FIVE SECTIONS – Section A, B, C, D and E.
iii) Each section is compulsory. However, there are internal choices in some questions.
iv) Section A has 18 MCQ'S and 02 Assertion-Reason based questions of 1 t each.
v) Section B has 5 Very Short Answer(VSA)-type questions of 2 marks each.
vi) Section C has 6 Short Answer(SA)-type questions of 3 marks each.
vii) Section D has 4 Long Answer(LA)-type questions of 5 marks each.
viii) Section E has 3 Case Based /Source Based questions of 4 marks each.

SECTION – A

1. If P is 3x3 matrix such that P' =2P + I, where P' is the transpose of P, then

a) P = I c) P = -I
b) P = 2I d) P = -2I

2. If A and B are square matrices of the same order 2 such that |A| = 2 and AB = 2I then the value of |B| is

a) 4 b) 1 c) 2 d) 8

3. If A and B are square matrices of the same order such that AB = 5I then A^{-1} is

a) 5B b) B/5 c) B d) $B^{-1}/5$

4. If the function $f(x) = \begin{cases} kx^2 + 5 & , x \le 1 \\ 2 & , x > 1 \end{cases}$ is continuous at x=1, then k is

a) -3 b) 3 c) 5 d) 2

5. The distance between the lines l_1 and l_2 given by
$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ is

a) $\frac{\sqrt{290}}{7}$ b) $\frac{\sqrt{293}}{7}$ c) $\frac{290}{7}$ d) $\frac{293}{7}$

6. In which of the following differential equations is the degree equal to its order?

a) $x^3\left(\frac{dy}{dx}\right) - \frac{d^3x}{dx^3} = 0$ b) $\left(\frac{d^3y}{dx^3}\right)^3 + sin\left(\frac{dy}{dx}\right) = 0$

c) $x^2(\frac{dy}{dx})^4 + siny - \left(\frac{d^2y}{dx^2}\right)^2 = 0$ d) $\left(\frac{dy}{dx}\right)^3 + x\left(\frac{d^2y}{dx^2}\right) - y^3\left(\frac{d^3y}{dx^3}\right) + y = 0$

7. If the minimum value of an objective function Z = ax + by occurs at two points (3, 4) and (4, 3) then

a) a + b = 0 b) a = b
c) 3a = b d) a = 3b

8. If $|\vec{a}| = 8, |\vec{b}| = 3\ and\ |\vec{a} \times \vec{b}| = 12, then\ value\ of\ \vec{a} \cdot \vec{b}\ is$

a) $6\sqrt{3}$ b) $8\sqrt{3}$ c) $12\sqrt{3}$ d) $\sqrt{3}$

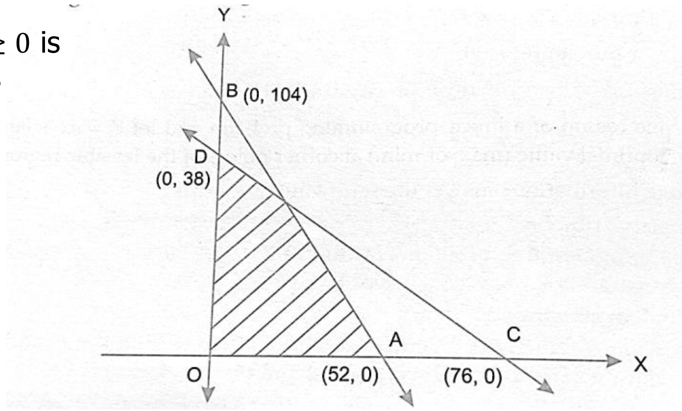
9. The value of $\int \frac{cos\sqrt{x}}{\sqrt{x}} dx$ is

a) $2\cos\sqrt{x} + C$ b) $\sqrt{\frac{cosx}{x}} + C$
c) $\sin\sqrt{x} + C$ d) $2\sin\sqrt{x} + C$

10. If A=$\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$, then $|A^2 - 2A|$ =

a) 5 b) 25 c) –5 d) –25

11. The feasible region of an LPP with $x, y \geq 0$ is given in the figure. Then the constraints of the LPP are



a) $2x + y \leq 52$ and $x + 2y \leq 76$ c) $x + 2y \leq 104$ and $2x + y \leq 76$
b) $2x + y \leq 104$ and $x + 2y \leq 76$ d) $x + 2y \leq 104$ and $2x + y \leq 38$

12. If the projection of $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on $\vec{b} = 2\hat{\imath} + \lambda\hat{k}$ is 0, then value of λ is
a) 0 b) 1 c) -2/3 d) -3/2

13. If $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ and $|A^3| = 27$, then the value of α is
a) ± 1 b) ± 2 c) $\pm\sqrt{5}$ d) $\pm\sqrt{7}$

14. If X is a random variable with probability distribution as given below:

X :	0	1	2	3
P(X):	k	3k	3k	k

then value of k is
a) 1/8 b) 1/2 c) 8 d) 1/3

15. The integrating factor of the differential equation $\tan^{-1} y\, dy - x dy = (1 + y^2) dx$ is
a) $\tan^{-1} x$ c) $e^{\tan^{-1} y}$
b) $e^{\tan^{-1} x}$ d) $\tan^{-1} y$

16. The direction cosines of a vector equally inclined to the axes OX, OY and OZ are
a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ c) 1, 1, 1
b) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ d) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

17. If $f(x) = \tan^{-1}(e^{2x})$ then $f'(x)$ is
a) $\frac{1}{1+e^{4x}}$ c) $\frac{2e^{2x}}{1+e^{4x}}$
b) $\frac{e^{2x}}{1+e^{4x}}$ d) $\frac{4e^{2x}}{1+e^{4x}}$

18. If $\vec{a}$ is any vector, then $(\vec{a} \times \hat{\imath})^2 + (\vec{a} \times \hat{\jmath})^2 + (\vec{a} \times \hat{k})^2 =$
a) $|\vec{a}|^2$ b) $2|\vec{a}|^2$ c) $3|\vec{a}|^2$ d) $4|\vec{a}|^2$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

a) Both (A) and (R) are true and (R) is the correct explanation of (A).
b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
c) (A) is true but (R) is false.
d) (A) is false but (R) is true.

19. Assertion (A): If x = at^2 and y = 2at then $\frac{d^2y}{dx^2} = -\frac{1}{2at^3}$.
Reason (R): $\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \times \frac{dt^2}{d^2x}$.

20. Assertion (A): Let A = {-2, -1, 0, 1, 2} and B = {0, 1, 4} where f: A→B given by f(x) = x^2 is a many-one function.
Reason (R): If $f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$ for every $x_1, x_2 \epsilon$ domain, then f is one-one or else many-one.

SECTION – B

21. Find the value of $cot\left\{cos^{-1}\frac{7}{25}\right\}$.

(OR)

Find the domain of $sin^{-1}\sqrt{x-1}$.

22. Find the intervals in which the function $f: R \to R$ defined by $f(x) = x - x^2$ is decreasing.

23. If $f(x) = -(x-1)^2 + 10$, x ∈ R then find the maximum value of f(x).

(OR)

The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue . If the total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue for x = 5.

24. Evaluate : $\int_0^{\pi/2} \log(\frac{4+3sinx}{4+3cosx})\, dx$.

25. Find the stationary point of a function $f(x) = x^x,\ x > 0$.

SECTION – C

26. Evaluate : $\int \frac{2x}{(x^2+1)(x^2+3)} dx$.

27. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

28. Evaluate : $\int \frac{dx}{\sqrt{5x^2-2X}}$ (OR) $\int_{-\pi/4}^{\pi/4} sin^2\, xdx$

29. Solve the Differential equation : $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2 \log x}$

(OR)

Solve the Differential Equation : $x\frac{dy}{dx} = y - x\sin(\frac{y}{x})$.

30. Solve the following LPP graphically.

Maximise Z = 3x + 9y
Such that x + 3y ≤ 60
x + y ≥ 10
x ≤ y
x, y ≥ 0

(OR)

Minimise Z = 50x + 70y
Such that 2x + y ≥ 8
x + 2y ≥ 10
x, y ≥ 0

31. If $(a+bx)e^{\frac{y}{x}} = x$ then prove $x\frac{d^2y}{dx^2} = (\frac{a}{a+bx})^2$.

SECTION – D

32. Find the area of the region bounded by the line y = 3x + 2, x-axis and the ordinates x = -1 and x = 1.

33. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

(OR)

Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that $f(x)$ is one-one and onto.

34. Using the matrix method, solve the following system of linear equations

2x + y + z = 1
x – 2y – z = 3/2
3y – 5z = 9

35. Find the vector equation of a line passing through the point (1, 1, 1) and perpendicular to the two lines $\vec{r} = -2\hat{\imath} + 3\hat{\jmath} - \hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$ and $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$.

(OR)

Find the equation of line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1).

SECTION – E

Case Study Based Questions / Source Based Integrated Questions.

Question nos. 36 and 37 have three subparts (i), (ii) and (iii) which carry 1, 1 and 2 marks respectively. Question 38 has two subparts of 2 marks each.

36. A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.
Find the probability that
i) All of them will hit the target.
ii) B and C will hit the target and A will lose.
iii) Any two will hit the target.

(OR)

Atleast one of A, B and C will hit the target.

37. Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively. Based on this information, answer the following questions:
i) Let $\vec{a},\ \vec{b}\ and\ \vec{c}$ be the position vectors of points A, B and C respectively, find $\vec{a} + \vec{b} + \vec{c}$.
ii) Find $\overrightarrow{AB}\ and\ \overrightarrow{AC}$.
iii) Find area of ΔABC.

(OR)

Find the unit vector in direction of vector $\vec{a}$.

38. The sum of surface areas of a cuboid $x,\ 2x\ and\ \frac{x}{3}$ and a sphere of radius r is given to be constant.
i) Find $\frac{dx}{dr}$.
ii) If volume is minimum, find relation between $x\ and\ r$.

-x-x-x-x-x-x-x-x-x-